模式分析与机器智能
工业和信息化部重点实验室
MIIT Key Laboratory of
Pattern Analysis & Machine Intelligence

ParNeC 模式识别与神经计算研究组
PAttern Recognition and NEural Computing

Deep Learning for Full Waveform Inversion

Background

ParNeC 模式识别与神经计算研究组
PAttern Recognition and NEural Computing

Full Waveform Inversion (FWI)

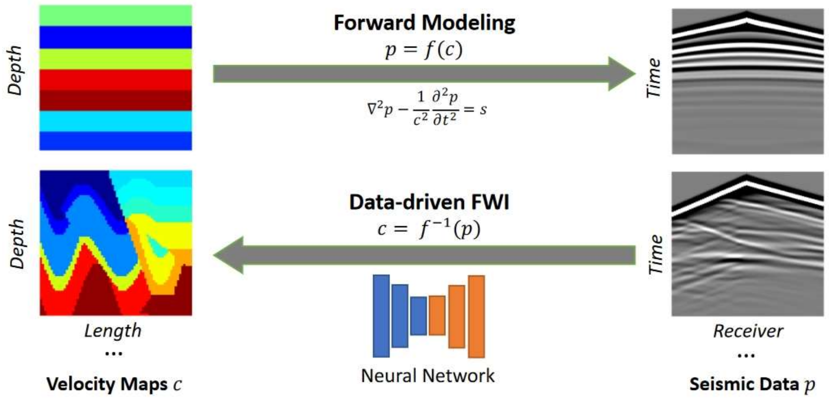


Figure 2: **Schematic illustration of data-driven FWI and forward modeling.** Neural networks are employed to infer velocity maps from seismic data while forward modeling is to calculate the seismic data using governing wave equations with velocity map provided.

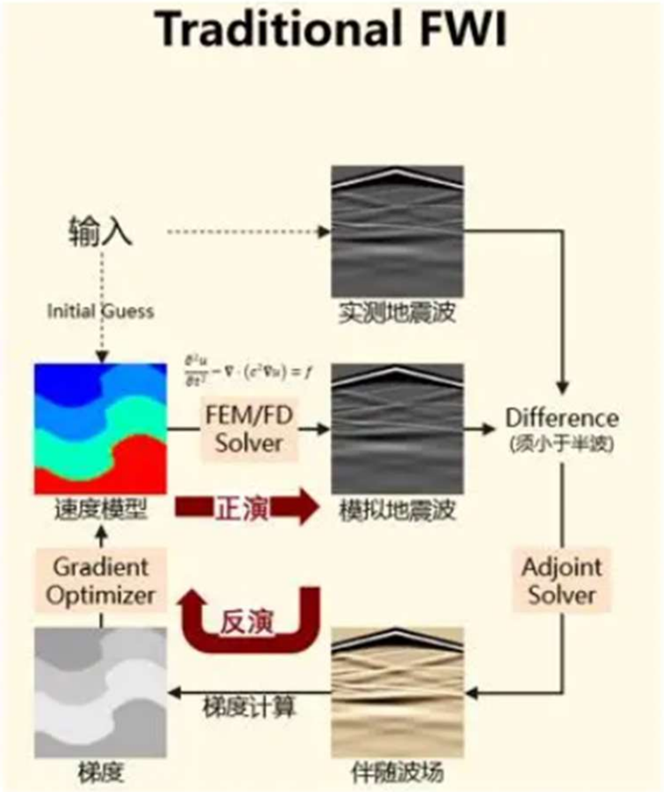
acoustic wave equation：

$$\nabla^2 p(x,z,t) - \frac{1}{c(x,z)^2}\frac{\partial^2 p(x,z,t)}{\partial t^2} = S_{xs,f}(x,z,t) \quad (1)$$

P: pressure wavefield
C: velocity model
S: source term

Background

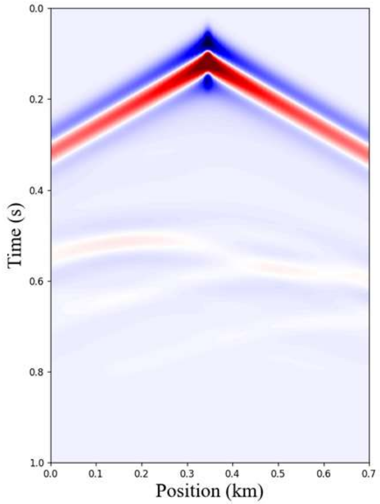
Traditional FWI



Dataset

OpenFWI

OPENFWI: Large-scale Multi-structural Benchmark Datasets for Full Waveform Inversion (neurips 2022)



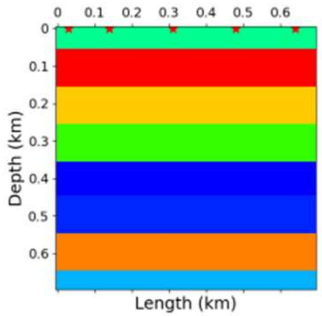


Table 2: **Dataset summary**. Explanation of data size: Velocity maps follow (depth × width × length); seismic data represents (#source × time × #receiver in width × #receiver in length).

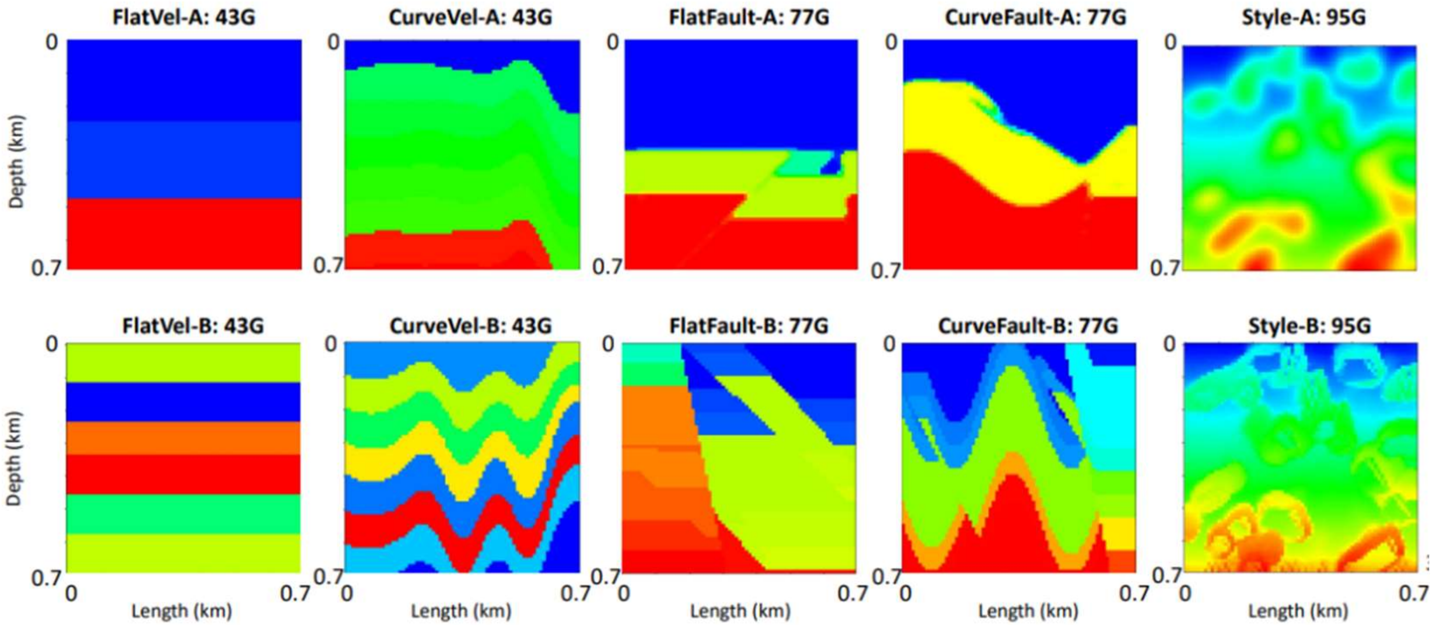
Group	Dataset	Size	#Train/#Test	Seismic Data Size	Velocity Map Size
Vel Family	FlatVel-A/B	43GB	24K / 6K	$5 \times 1000 \times 1 \times 70$	$70 \times 1 \times 70$
	CurveVel-A/B	43GB	24K / 6K	$5 \times 1000 \times 1 \times 70$	$70 \times 1 \times 70$
Fault Family	FlatFault-A/B	77GB	48K / 6K	$5 \times 1000 \times 1 \times 70$	$70 \times 1 \times 70$
	CurveFault-A/B	77GB	48K / 6K	$5 \times 1000 \times 1 \times 70$	$70 \times 1 \times 70$
Style Family	Style-A/B	95GB	60K / 7K	$5 \times 1000 \times 1 \times 70$	$70 \times 1 \times 70$
Kimberlina Family	Kimberlina-CO_2	96GB	15K / 4430	$9 \times 1251 \times 1 \times 101$	$141 \times 1 \times 401$
	3D Kimberlina-V1	1.4TB	1664 / 163	$25 \times 5001 \times 40 \times 40$	$350 \times 400 \times 400$

Table 3: **Physical Meaning of OpenFWI dataset**

Dataset	Grid Spacing	Velocity Map Spatial Size	Source Spacing	Source Line Length	Receiver Line Spacing	Receiver Line Length	Time Spacing	Recorded Time
"Vel, Fault and Style" Family	$10\ m$	$0.7 \times 0.7\ km^2$	$140\ m$	$0.7\ km$	$10\ m$	$0.7\ km$	$0.001\ s$	$1\ s$
Kimberlina-CO_2	$10\ m$	$1.4 \times 4\ km^2$	$400\ m$	$3.6\ km$	$40\ m$	$4\ km$	$0.002\ s$	$2.5\ s$
3D Kimberlina-V1	$10\ m$	$3.5 \times 4 \times 4\ km^3$	$800\ m$	$(4\ km, 4\ km)$	$100\ m$	$(4\ km, 4\ km)$	$0.001\ s$	$5\ s$

OpenFWI

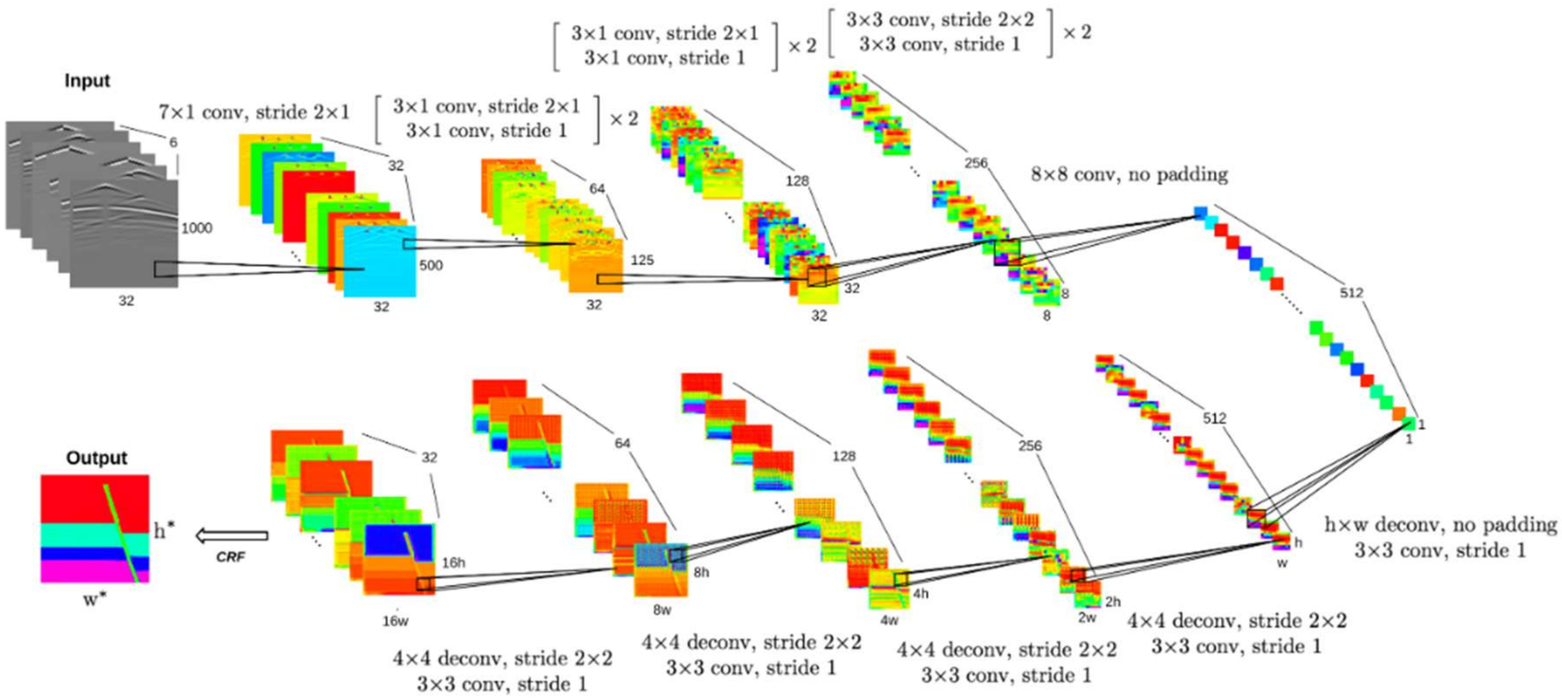
OPENFWI: Large-scale Multi-structural Benchmark Datasets for Full Waveform Inversion (neurips 2022)



Deep Learning for FWI

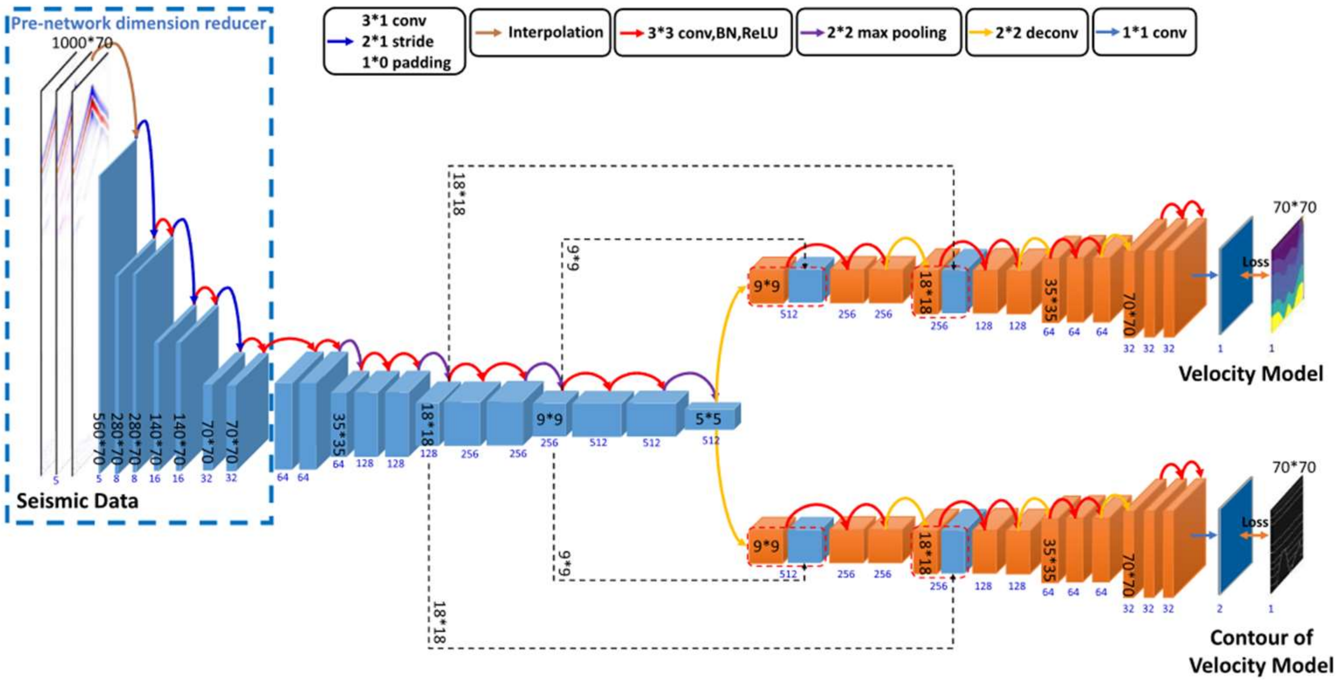
InversionNet

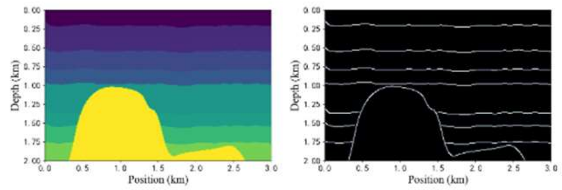
InversionNet: An efficient and accurate data-driven full waveform inversion
(IEEE Transactions on Computational Imaging 2019)



DD-Net

DD-Net: Dual decoder network with curriculum learning for full waveform inversion
(IEEE Transactions on Geoscience and Remote Sensing 2024)





(a) (b)

Fig. 3. Example of a velocity model and its contour. (a) Simulated velocity model. (b) Contour structure of this velocity model obtained by the Canny [41]. The dual thresholds of Canny are set to 10 and 15, respectively.

Deep Learning for FWI

ParNeC 模式识别与神经计算研究组 PAttern Recognition and NEural Computing

Vanilla DeepONet

Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators (Nature 2021)

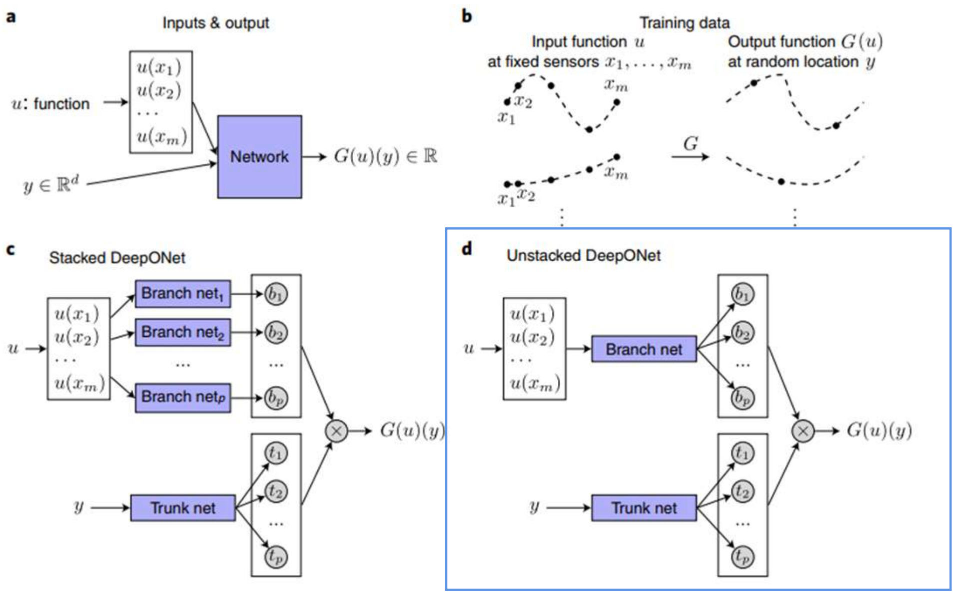


Fig. 1 | Illustrations of the problem set-up and new architectures of DeepONets that lead to good generalization. a, For the network to learn an operator $G : u \mapsto G(u)$ it takes two inputs $[u(x_1), u(x_2), \ldots, u(x_m)]$ and y. **b**, Illustration of the training data. For each input function u, we require that we have the same number of evaluations at the same scattered sensors $x_1, x_2, \ldots, x_m$. However, we do not enforce any constraints on the number or locations for the evaluation of output functions. **c**, The stacked DeepONet is inspired by Theorem 1, and has one trunk network and p stacked branch networks. The network constructed in Theorem 1 is a stacked DeepONet formed by choosing the trunk net as a one-layer network of width p and each branch net as a one-hidden-layer network of width n. **d**, The unstacked DeepONet is inspired by Theorem 2, and has one trunk network and one branch network. An unstacked DeepONet can be viewed as a stacked DeepONet with all the branch nets sharing the same set of parameters.

Deep Learning for FWI

ParNeC 模式识别与神经计算研究组 PAttern Recognition and NEural Computing

Fourier-DeepONet

Fourier-DeepONet: Fourier-enhanced deep operator networks for full waveform inversion with improved accuracy, generalizability, and robustness
(Computer Methods in Applied Mechanics and Engineering 2023)

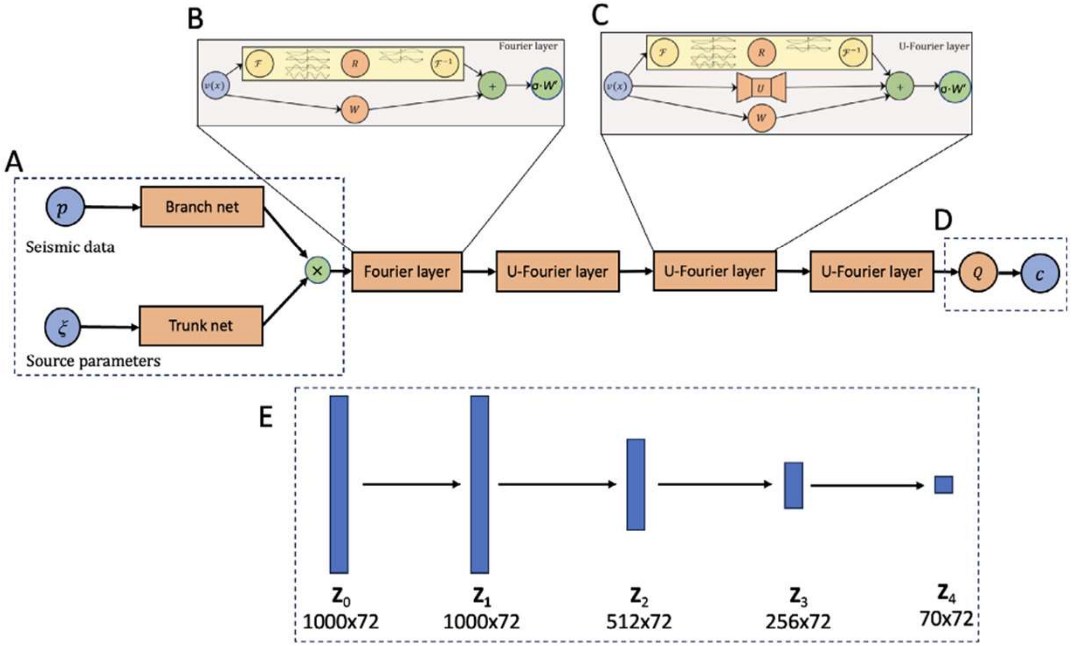


Fig. 3. Fourier-DeepONet architecture. (A) Branch net and trunk net are two linear transformations lifting inputs to high dimensional space. Green circle represents the merger operation which denotes point-wise multiplication. (B) Fourier layer, adapted from [39]. (C) U-Fourier layer, adapted from [38]. (D) Projection layer Q. (E) Shapes of outputs $\mathbf{z}_i$ in one channel, $i \in \{0, 1, 2, 3, 4\}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Fourier-DeepONet architecture. The merger net includes one Fourier layer and three U-Fourier layers. The output shape does not include the batch size dimension.

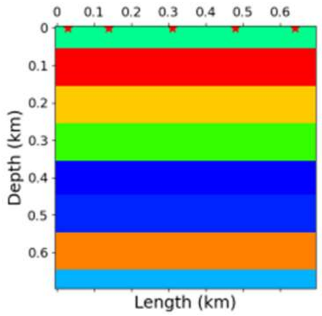
	Operations	Output shape
Branch net	Padding, Linear	(1000, 72, 64)
Trunk net	Linear	(64)
Merger operation	Pointwise multiplication of branch and trunk outputs	(1000, 72, 64)
Fourier	Add(Fourier2d+Conv1d), ReLU	(1000, 72, 64)
U-Fourier 1	Add(Fourier2d+Conv1d+UNet2d), Linear, ReLU	(512, 72, 64)
U-Fourier 2	Add(Fourier2d+Conv1d+UNet2d), Linear, ReLU	(256, 72, 64)
U-Fourier 3	Add(Fourier2d+Conv1d+UNet2d), Linear, ReLU	(70, 72, 64)
Projection	Linear, ReLU, Linear, Slicing	(70, 70)

Deep Learning for FWI

Fourier-DeepONet

Source frequencies and locations of different datasets. The source frequency and location in OpenFWI are fixed, whereas in FWI-F, FWI-L, and FWI-FL, the source frequencies, locations, and both frequencies and locations vary, respectively.

Dataset	Frequency (Hz)	Location (m)				
		Source A	Source B	Source C	Source D	Source E
OpenFWI	15	0	172.5	345	517.5	690
FWI-F	[5, 25]	0	172.5	345	517.5	690
FWI-L	15	[0, 50]	[122.5, 222.5]	[295, 295]	[467.5, 567.5]	[640, 690]
FWI-FL	[5, 25]	[0, 50]	[122.5, 222.5]	[295, 295]	[467.5, 567.5]	[640, 690]

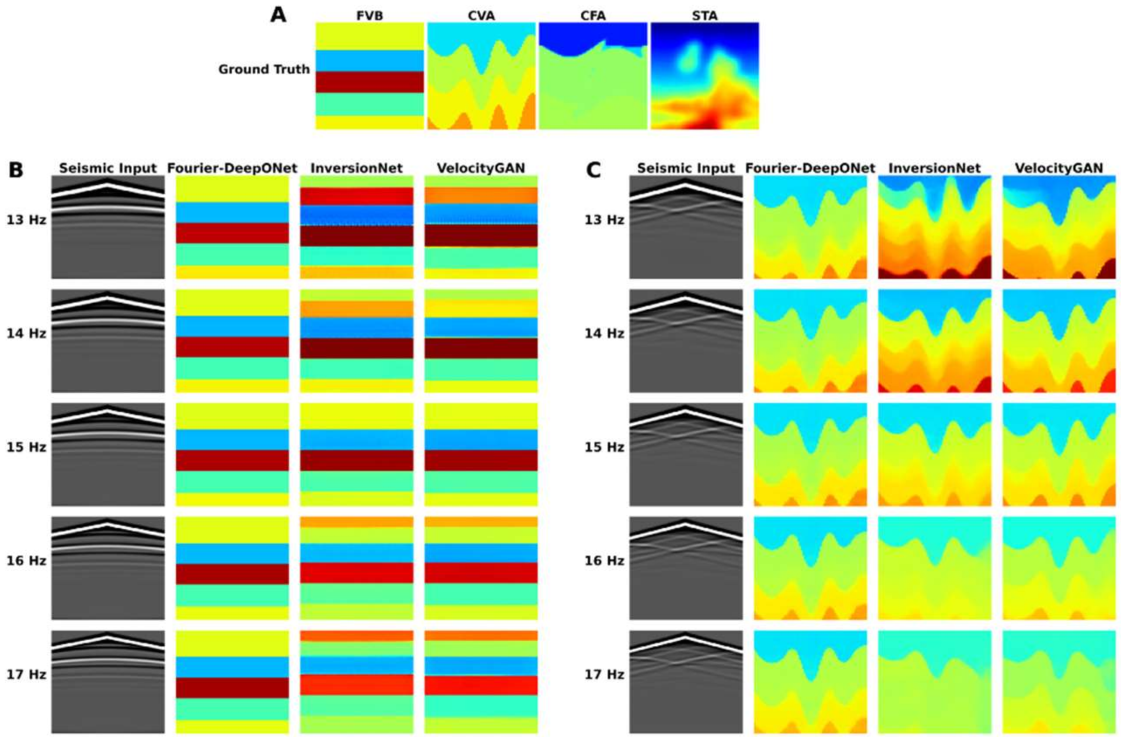


- FWI-F: The source frequency is ranging from 5 to 25 Hz.
- FWI-L: We allow the middle three sources (B, C, and D) to move up to 50 m either to the left or the right, while the sources at the boundaries (A and E) can move towards the center. Hence, the possible horizontal coordinates of the five sources are [0, 50], [122.5, 222.5], [295, 295], [467.5, 567.5], and [640, 690] m, and the source spacing varies between 72.5 and 272.5 m.
- FWI-FL: The source frequency and location follow the patterns of FWI-F and FWI-L, respectively.

Deep Learning for FWI

ParNeC 模式识别与神经计算研究组 PAttern Recognition and NEural Computing

Fourier-DeepONet



Enhanced Datast

Source frequencies and locations of different datasets. The source frequency and location in OpenFWI are fixed, whereas in FWI-F, FWI-L, and FWI-FL, the source frequencies, locations, and both frequencies and locations vary, respectively.

Zhu et al. 2023 Fourier-DeepONet

Dataset	Frequency (Hz)	Location (m) Source A	Source B	Source C	Source D	Source E
OpenFWI	15	0	172.5	345	517.5	690
FWI-F	[5, 25]	0	172.5	345	517.5	690
FWI-L	15	[0, 50]	[122.5, 222.5]	[295, 295]	[467.5, 567.5]	[640, 690]
FWI-FL	[5, 25]	[0, 50]	[122.5, 222.5]	[295, 295]	[467.5, 567.5]	[640, 690]

Ours

Dataset		Source A	Source B	Source C	Source D	Source E
OpenFWI	frequency	15	15	15	15	15
	location	0	172.5	345	517.5	690
FWI-F	frequency	[5, 25]	[5, 25]	[5, 25]	[5, 25]	[5, 25]
	location	0	172.5	345	517.5	690
FWI-L	frequency	15	15	15	15	15
	location	[0, 50]	[122.5, 222.5]	[295, 395]	[467.5, 567.5]	[640, 690]
FWI-FL	frequency	[5, 25]	[5, 25]	[5, 25]	[5, 25]	[5, 25]
	location	[0, 50]	[122.5, 222.5]	[295, 395]	[467.5, 567.5]	[640, 690]

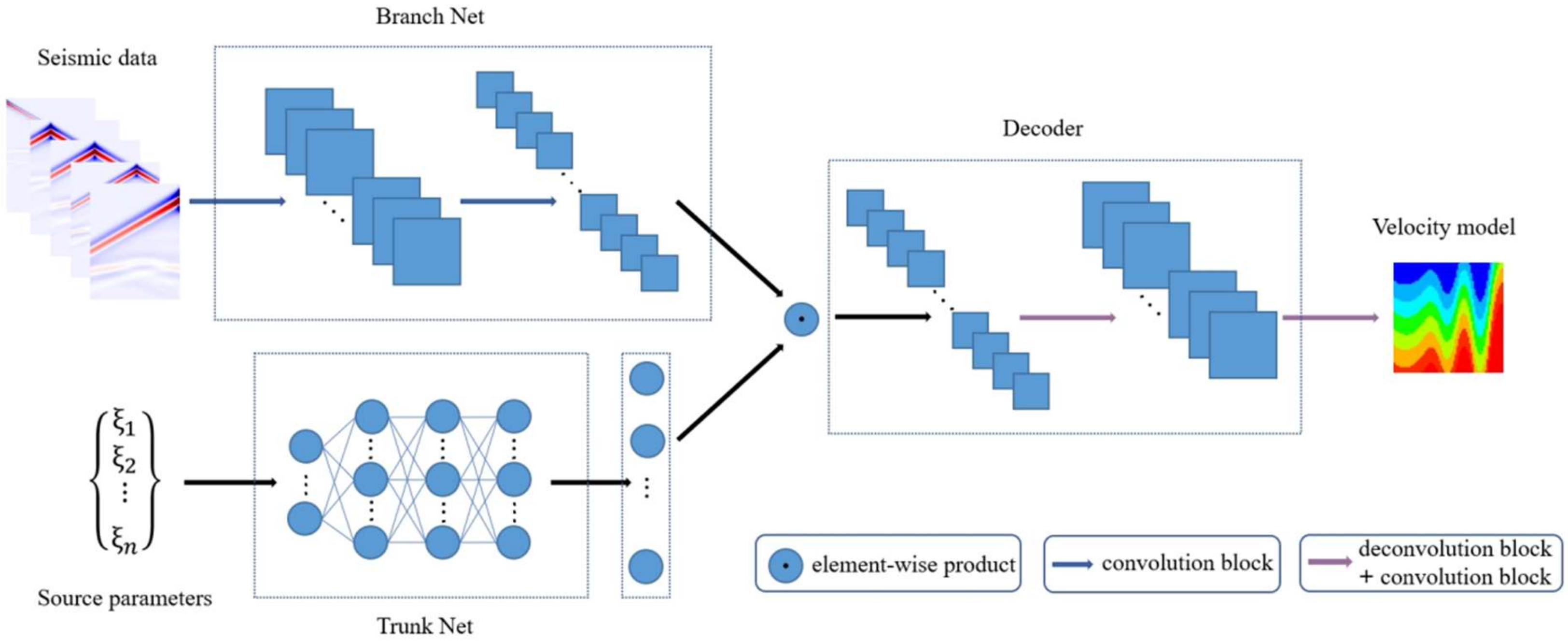
Table 1: Detailed description of our enhanced datasets. The obvious distinction is that the five sources simultaneously have different frequencies varying from 5 to 25 Hz in our datasets, while the frequencies are the same in (Zhu et al. 2023).

Low frequencies provide deeper penetration which help in resolving larger-scale structures and provide information about the deeper parts of the subsurface.

High Frequencies offer higher resolution and are more sensitive to smaller-scale features which help in imaging finer details of the subsurface.

My Work

Our method



Comparison of encoder-decoder structures

Encoder-Decoder	MAE	RMSE	SSIM	RE	Batch Size	Speed (s/epoch)
InverisonNet	**0.0591**	**0.0143**	**0.8262**	**0.2136**	128	62
Fourier-DeepONet	0.0717	0.0176	0.7984	0.2393	32	525
DD-Net70	0.0828	0.0199	0.7749	0.2558	128	50
Vanilla DeepONet	0.1100	0.0273	0.7248	0.3141	128	53

Table 4: The quantitative results of the four encoder-decoder structures tested on CurveVel-A in FWI-F. The encoder-decoder of InversionNet, Fourier-DeepONet and DD-Net70 are employed in novel DeepONet architecture. For DD-Net70, we just use the first decoder. For Vanilla DeepONet, the coordinate matrix, which are positions corresponding to every velocity value in the subsurface, are fed into trunk net.

Experment

Performance on our dataset

Dataset	Model	MAE	RMSE	SSIM	RE
FlatVel-B	InversionNet	0.0781	0.0336	0.8354	0.2756
	Fourier-DeepONet	0.0637	0.0244	0.8602	0.2281
	Our Method	**0.0525**	**0.0200**	**0.8866**	**0.1979**
CurveVel-A	InversionNet	0.0861	0.0219	0.7596	0.2697
	Fourier-DeepONet	0.0717	0.0176	0.7984	0.2393
	Our Method	**0.0591**	**0.0143**	**0.8262**	**0.2136**
FlatFault-B	InversionNet	0.1118	0.0314	0.7141	0.3088
	Fourier-DeepONet	0.1155	0.0329	0.7093	0.3175
	Our Method	**0.1021**	**0.0294**	**0.7240**	**0.2955**
CurveFault-A	InversionNet	0.0364	0.0078	0.9285	0.1493
	Fourier-DeepONet	0.0411	0.0096	0.9152	0.1670
	Our Method	**0.0269**	**0.0058**	**0.9414**	**0.1255**

Table 3: The quantitative results of InversionNet, Fourier-DeepONet and Inversion-DeepONet tested on FWI-F. We validate those methods on four categories of velocity models, which have five sources with fixed locations and different frequencies between 5 and 25 Hz. The Inversion-DeepONet has the best performance under all metrics.

Experment

Performance on our dataset

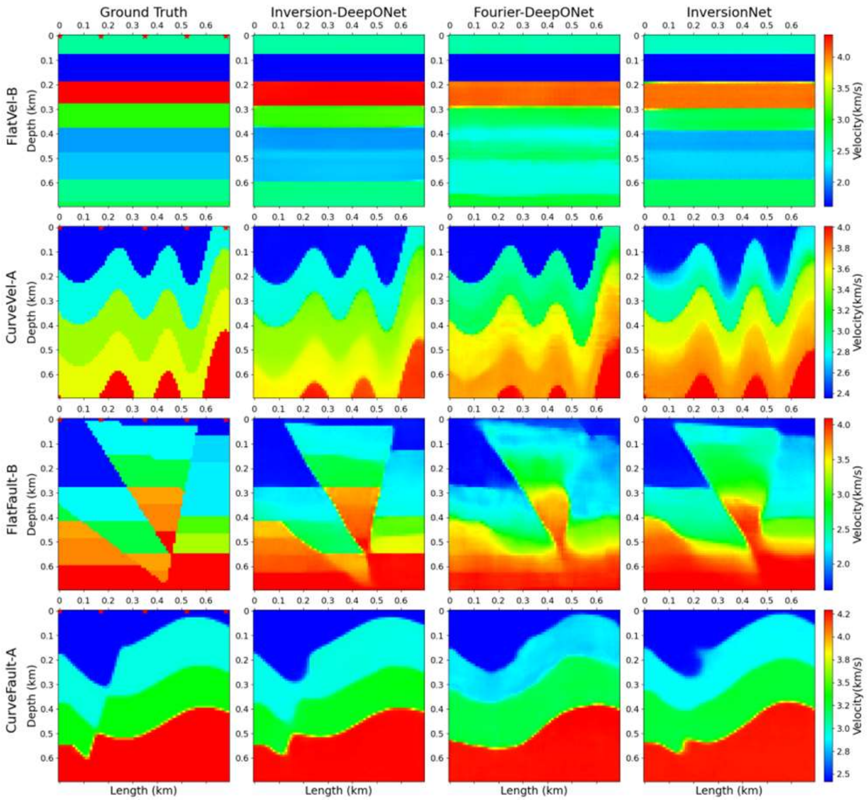


Figure 6: Examples of inversion results predicted by InversionNet-DeepONet, Fourier-DeepONet and InversionNet tested on four categories of velocity models from FWI-F. The five red stars in the ground truth are sources, which have different frequencies.

Experment

Generalization analysis

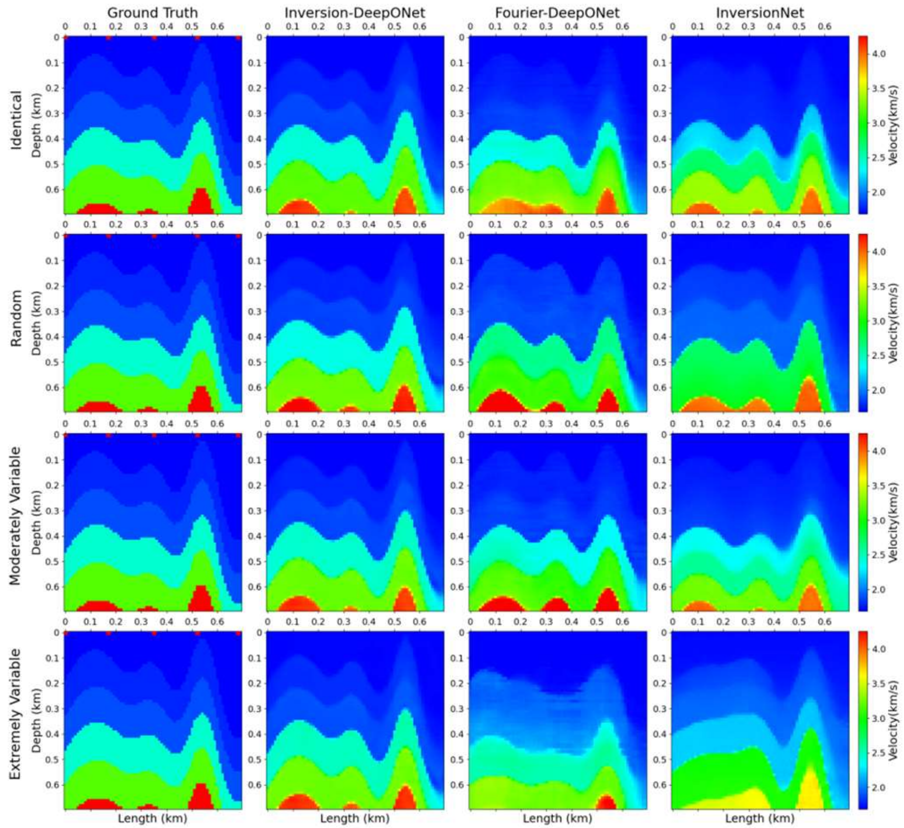


Figure 5: The generalization ability of Inversion-DeepONet, Fourier-DeepONet and InversionNet tested on CurveVel-A in FWI-F. Examples of different scenarios of five source frequencies: (1) Identical at 15 Hz. (2) Random at 19.9, 9.7, 10.0, 16.4 and 5.2 Hz. (3) Moderately variable at 10, 15, 20, 15 and 10 Hz. (4) Extremely variable at 5, 15, 25, 15 and 5 Hz.

Thanks