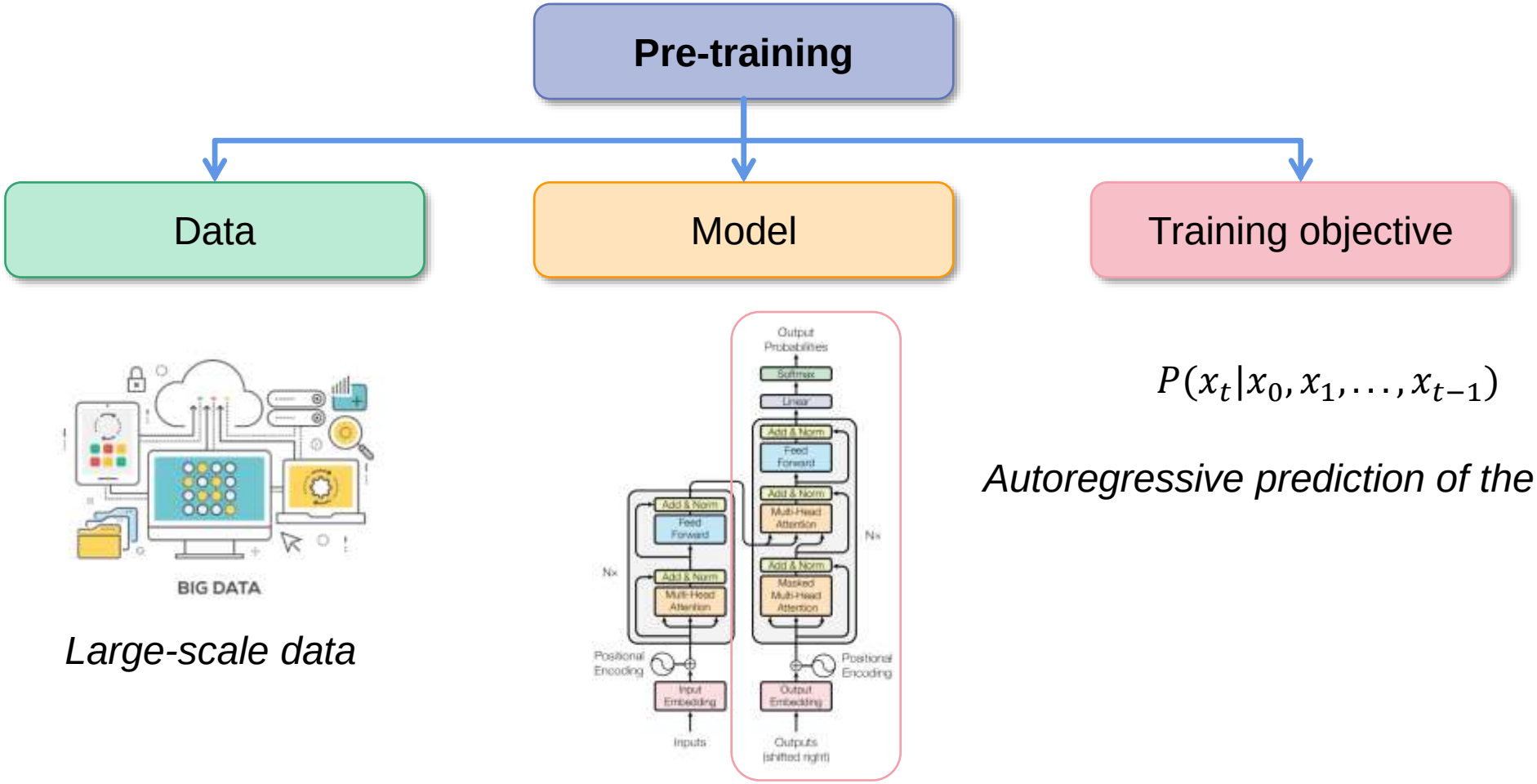
RL in LLMs

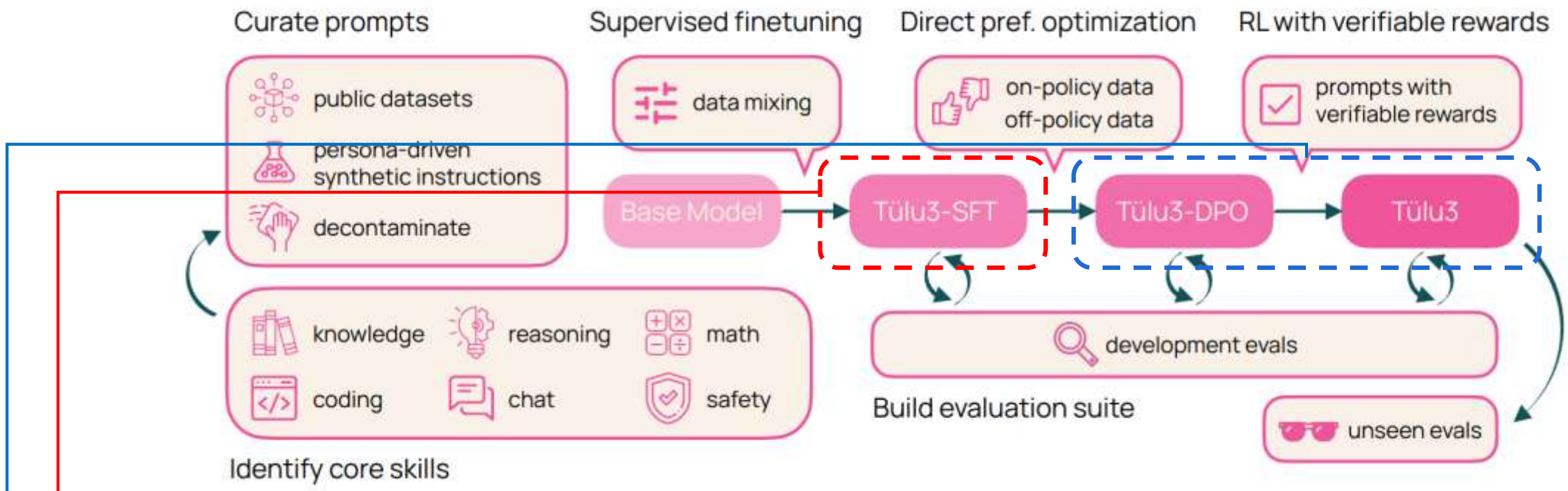
Pre-training



✓ Build a **base model** with general language understanding and generation capabilities.

Post-training

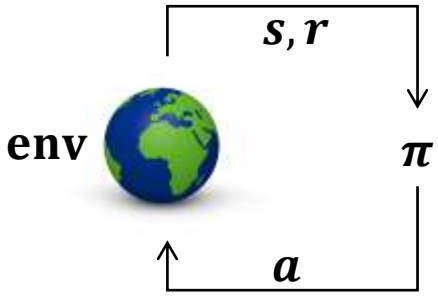
Tülu3



1. **Supervised Fine-Tuing (SFT)**: use data with annotations to build the ability of *instruction following*
2. **Reinforcement Learning (RL)**:
   - optimize the model behavior to *align with human preferences*
   - promote the evolution of model *autonomous reasoning ability*

Background - RL

Concept



- env → s_0
- $\pi \rightarrow a_0 \sim \pi(\cdot|s_0)$
- env → s_1, r_0
- ...

MDP(Markov Decision Process)

Definition: (S, A, R, P, γ)

$s \in S \quad a \in A \quad r_t = R(s_t, a_t)$

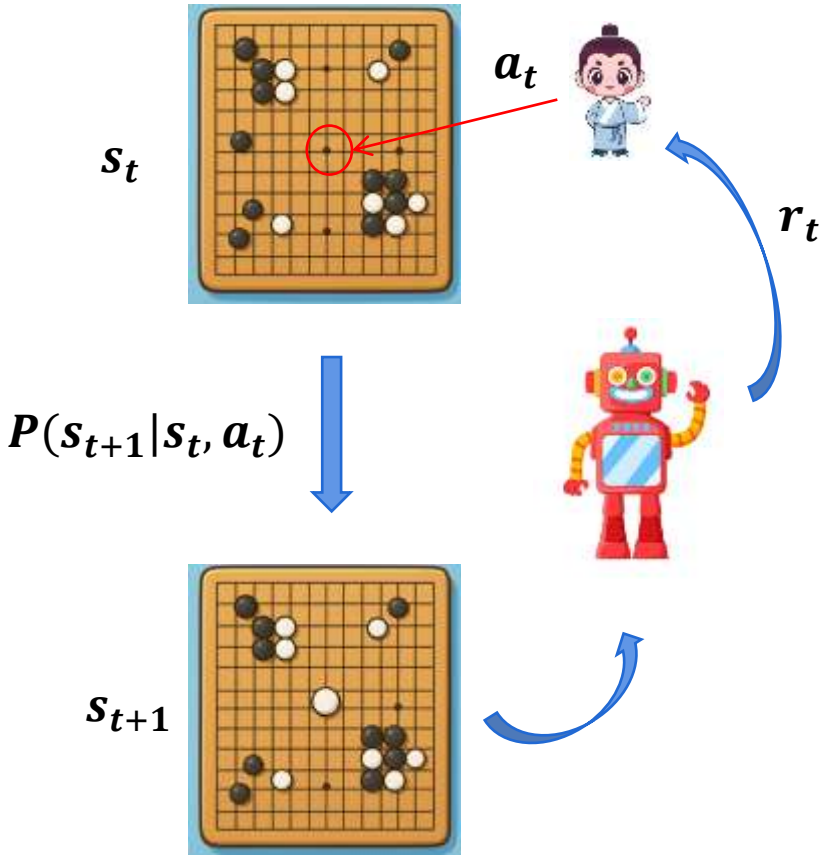
Markov property: $P(s_{t+1}|s_{<t}, a_{<t}, s_t, a_t)$

$P(s_{t+1}|s_t, a_t)$

Policy objective:

$$J(\pi) = \mathbb{E}_{a_0 \sim \pi(\cdot|s_0)}\left[r(s_0, a_0) + \gamma \mathbb{E}_{s_1 \sim P(\cdot|s_0, a_0)}\left[\mathbb{E}_{a_1 \sim \pi(\cdot|s_1)}[r(s_1, a_1) + \cdots]\right]\right]$$

Example



RL for LLMs

MDP: (S, A, R, P, γ) ⟹ Language-augmented MDP: (V, S, A, R, P, γ)

V: vocabulary A specific token: $w \in V$

S: $S \subset V^M$ $s = (w_1, w_2, w_3, \ldots, w_M)$ s_0: prompt

A: $S \subset V^N$ $a = (w_1, w_2, \ldots, w_N)$

R: $r = R(s, a)$

P: $S \times V \to S$ $s_{i+1} = (s_i, w_i) = (s_0, w_{1:i+1})$ auto-regressive paradigm

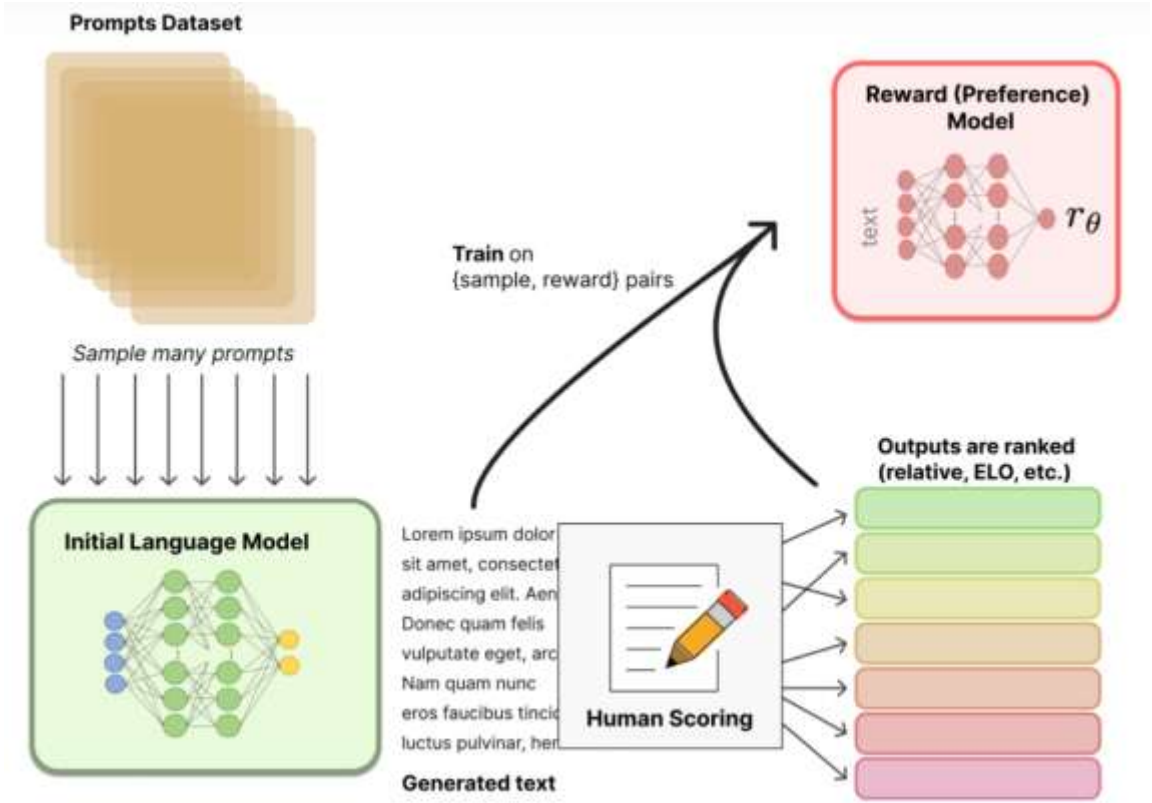
Token-level policy: $\pi(w_i | s_0, w_{1:i-1})$

Sentence-level policy: $\pi(a|s_0) = \prod_{i=1}^{N} \pi(w_i | s_0, w_{1:i-1})$

Policy objective:

$$J(\pi) = \mathbb{E}_{s_0 \sim D}\left[\mathbb{E}_{a \sim \pi(\cdot|s_0)}[r(s_0, a)]\right]$$

Why use RL?



Prompt: s
Candidate answers: a_1, a_2, a_3, a_4
Quality rank: $a_1 > a_2 > a_3 > a_4$

How can we optmize the model?

Supervised learning: $\mathbf{max}\ \mathbf{log}\ \boldsymbol{p}(\boldsymbol{a_1}|\boldsymbol{s})$

⇩

Data waste & poor generalization

A solution:

for each answers: $\mathbf{max}\ \ \boldsymbol{R_i} \cdot \mathbf{log}\ \boldsymbol{p}(\boldsymbol{a_i}|\boldsymbol{s})$

gradient: $\boldsymbol{R_i} \cdot \boldsymbol{\nabla_\theta} \mathbf{log}\ \boldsymbol{p_\theta}(\boldsymbol{a_i}|\boldsymbol{s})$

↓

✓ **The policy gradient of RL**

Policy Gradient

The goal of policy in RL: $J(\pi_\theta) = \mathbb{E}_{\tau\sim\pi_\theta}[R(\tau)]$

The policy gradient: $\nabla_\theta J(\pi_\theta) = \mathbb{E}_{\tau\sim\pi_\theta}[\sum_{t=0}^{T} \nabla_\theta \log\pi_\theta(a_t|s_t) R_t]$, where $R_t = \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'}$

Single Step $\nabla_\theta J(\pi_\theta) = \mathbb{E}_{s_0\sim D, a\sim\pi_\theta(\cdot|s_0)}[\boldsymbol{r(s_0,a)\nabla_\theta \log \pi_\theta(a|s_0)}]$ $\longleftrightarrow$ $\boldsymbol{R_i \cdot \nabla_\theta \log p_\theta(a_i|s)}$

or $\boldsymbol{A(s_0,a)} = r(s_0,a) - b$

The goal of RL in LLMs is to fine-tune the pretrained model to maximize the probability of high-quality responses

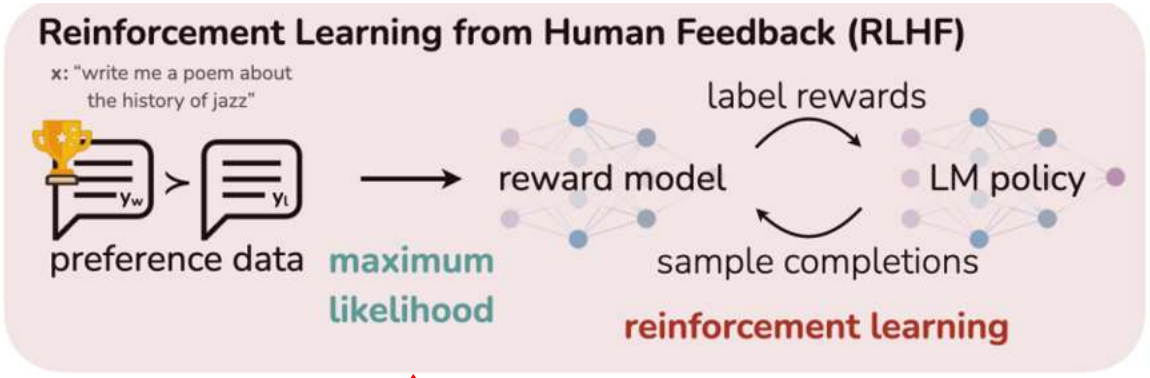
$$J(\pi_\theta) = \mathbb{E}_{s_0\sim D}\left[\mathbb{E}_{a\sim\pi_\theta(\cdot|s_0)}[r(s_0,a)] - \boldsymbol{\beta D_{KL}(\pi_\theta, \pi_{ref})}\right]$$

Sentence-level $R(s_0,a) = r(s_0,a) - \beta \log \frac{\pi_\theta(a|s_0)}{\pi_{ref}(a|s_0)}$

token-level

$$R(s_0,w_i) = \begin{cases} -\beta \sum_{i=1}^{T} \log \frac{\pi_\theta(w_i|s_0)}{\pi_{ref}(w_i|s_0)} & T < N \\ r(s_0,a) - \sum_{i=1}^{N} \log \frac{\pi_\theta(w_i|s_0)}{\pi_{ref}(w_i|s_0)} & T = N \end{cases}$$

The Evolution of RL in LLMs



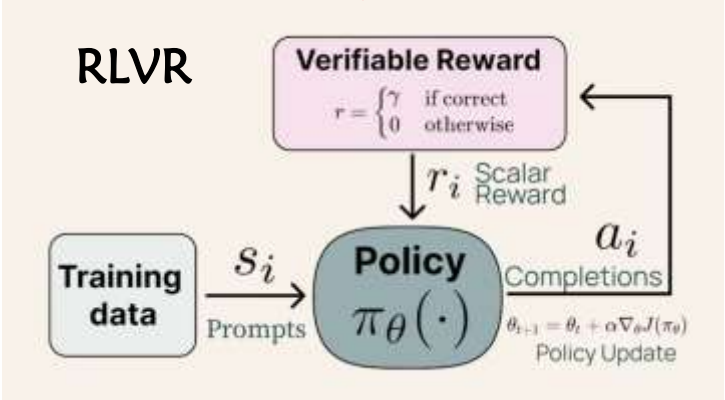
Optimize model to **align human preference** based on the answers with **manually annotated preferences**

?

GPT-4/Deepseek-R1
2024

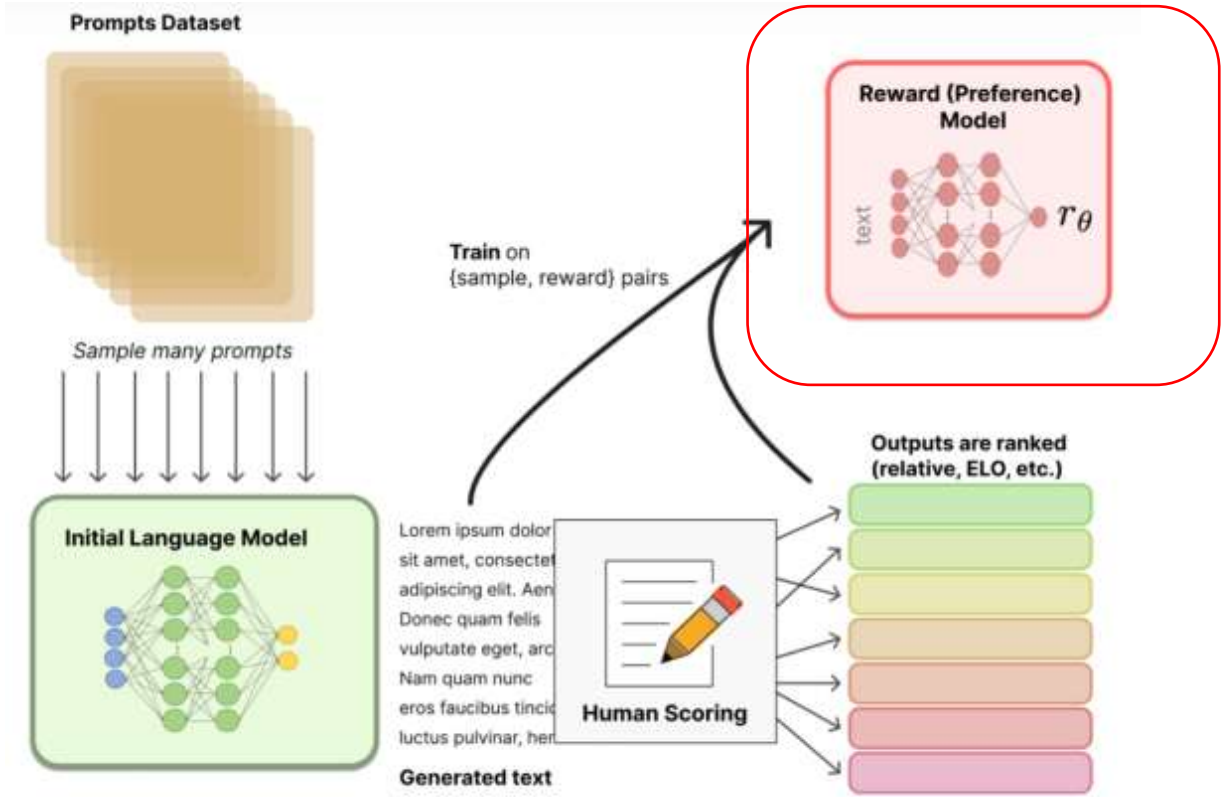
InstructGPT/ChatGPT
2022

Timeline



Automatically **verify the answers** to obtain rewards

The First Stage - RLHF



Preference-based reward:

$$\hat{P}(a_c > a_r|s) = \frac{exp[\hat{r}(s,a_c)]}{\exp[\hat{r}(s,a_c)] + exp[\hat{r}(s,a_r)]}$$ Bradley-Terry (BT) model

CE loss

$$L(\hat{r}) = -\mathbb{E}_{(s,a_c,a_r)\sim D}[\log \hat{P}(a_c > a_r|s)]$$

$$= -\mathbb{E}_{(s,a_c,a_r)\sim D}[\log(\sigma(\hat{r}(s,a_c) - \hat{r}(s,a_r)))]$$

Algorithm - PPO

$$\max_{\pi_\theta} \mathbb{E}_s \, \mathbb{E}_{a\sim\pi_{old}(\cdot|s)} \left[\frac{\pi_\theta(a|s)}{\pi_{old}(a|s)} A^{\pi_{old}}(s,a) \right]$$

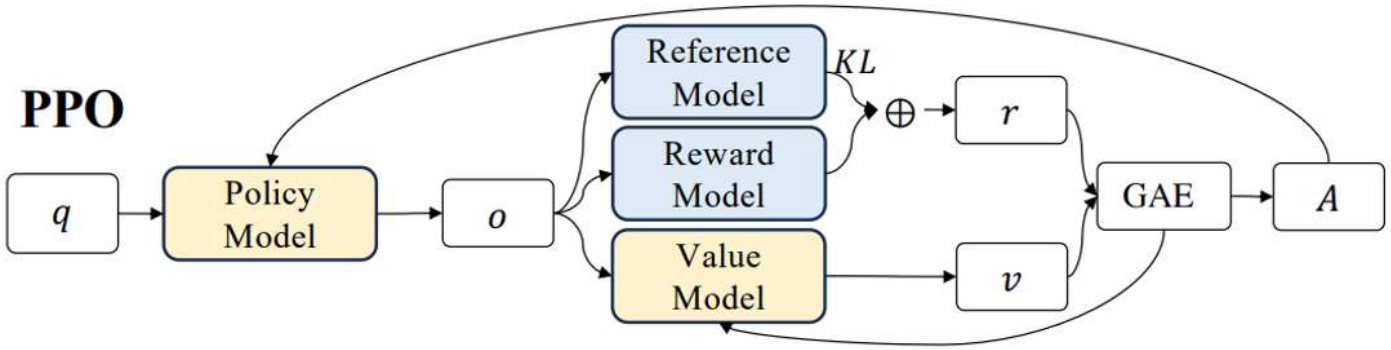
importance sampling

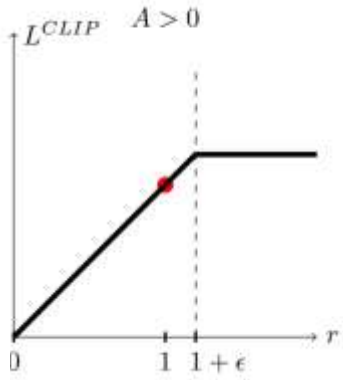
estimated based on a value model

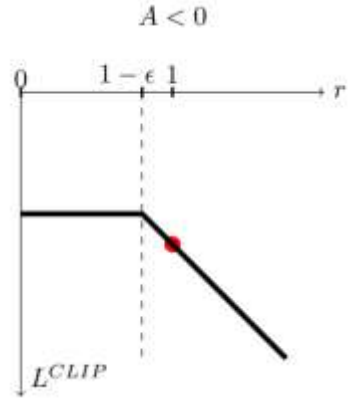
$$\text{s.t.} \quad \mathbb{E}_s[D_{KL}(\pi_{old}(a|s), \pi_\theta(\cdot|s))] < \delta$$

$$\operatorname{argmax}_\theta \mathbb{E}_s \, \mathbb{E}_{a\sim\pi_{old}(\cdot|s)} \left[\min \left(\frac{\pi_\theta(a|s)}{\pi_{old}(a|s)} A^{\pi_{old}}(s,a), \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{old}(a|s)}, 1-\epsilon, 1+\epsilon \right) A^{\pi_{old}}(s,a) \right) \right]$$

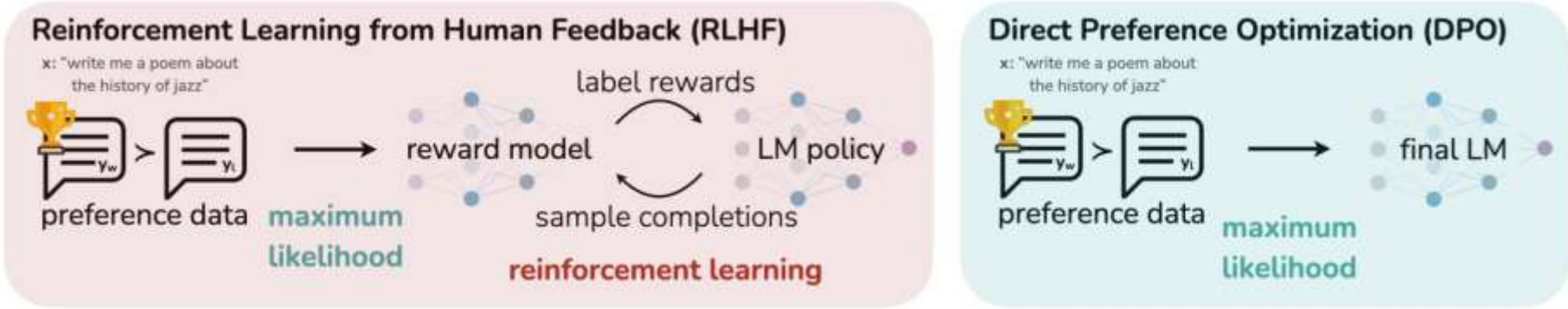
✓ **restricts how much the model policy can change at each step**







Algorithm - DPO



- rewrite the objective

$$\max_{\pi} \mathbb{E}_{x\sim\mathcal{D},y\sim\pi}\left[r(x,y)\right] - \beta\mathbb{D}_{\mathrm{KL}}\left[\pi(y|x)\,||\,\pi_{\mathrm{ref}}(y|x)\right]$$

$$\min_{\pi} \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log \frac{\pi(y|x)}{\frac{1}{Z(x)}\pi_{\mathrm{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)} - \log Z(x)\right] \quad Z(x) = \sum_{y}\pi_{\mathrm{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)$$

- explicit reward

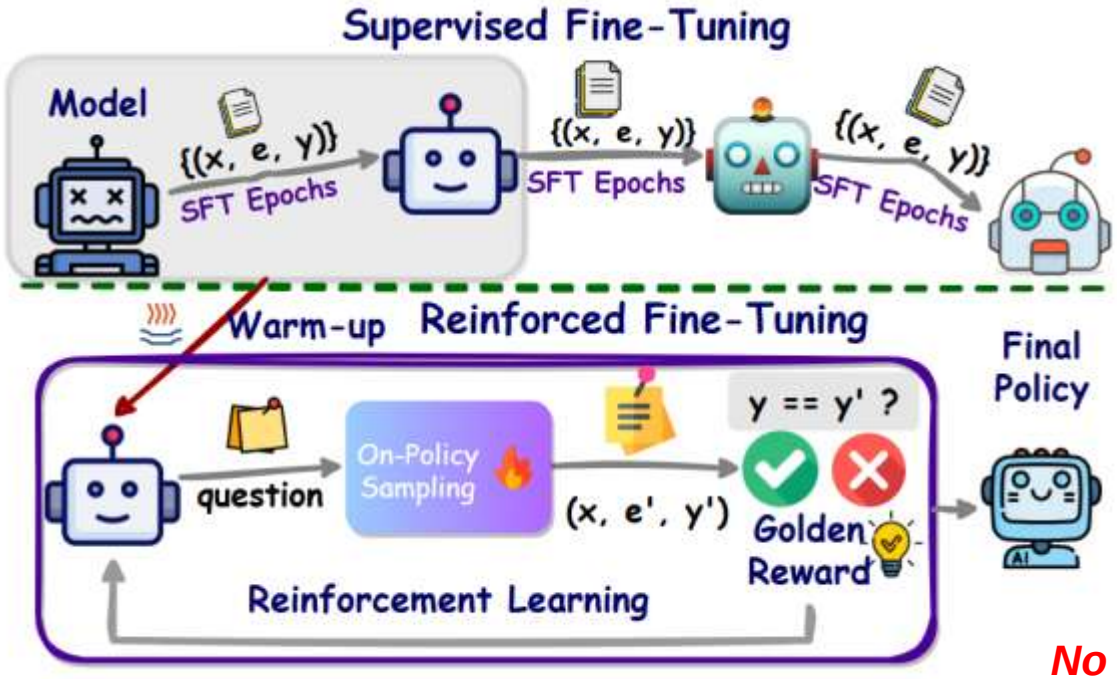
$$\pi^*(y|x) = \frac{1}{Z(x)}\pi_{\mathrm{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right) \rightarrow r^*(x,y) = \beta\log\frac{\pi^*(y|x)}{\pi_{\mathrm{ref}}(y|x)} + \beta\log Z(x)$$

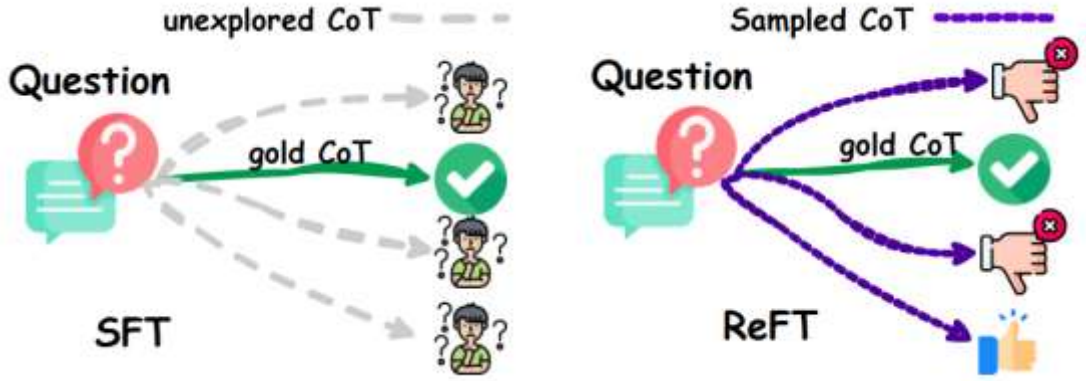
- BT model with explicit reward

$$p^*(y_1 \succ y_2 \mid x) = \frac{1}{1+\exp\left(\beta\log\frac{\pi^*(y_2|x)}{\pi_{\mathrm{ref}}(y_2|x)} - \beta\log\frac{\pi^*(y_1|x)}{\pi_{\mathrm{ref}}(y_1|x)}\right)}$$

$$\mathcal{L}_{\mathrm{DPO}}(\pi_\theta;\pi_{\mathrm{ref}}) = -\mathbb{E}_{(x,y_w,y_l)\sim\mathcal{D}}\left[\log\sigma\left(\beta\log\frac{\pi_\theta(y_w\mid x)}{\pi_{\mathrm{ref}}(y_w\mid x)} - \beta\log\frac{\pi_\theta(y_l\mid x)}{\pi_{\mathrm{ref}}(y_l\mid x)}\right)\right]$$

The Secode Stage - RLVR





No human in the loop

Rule-based (Verifiable) reward:

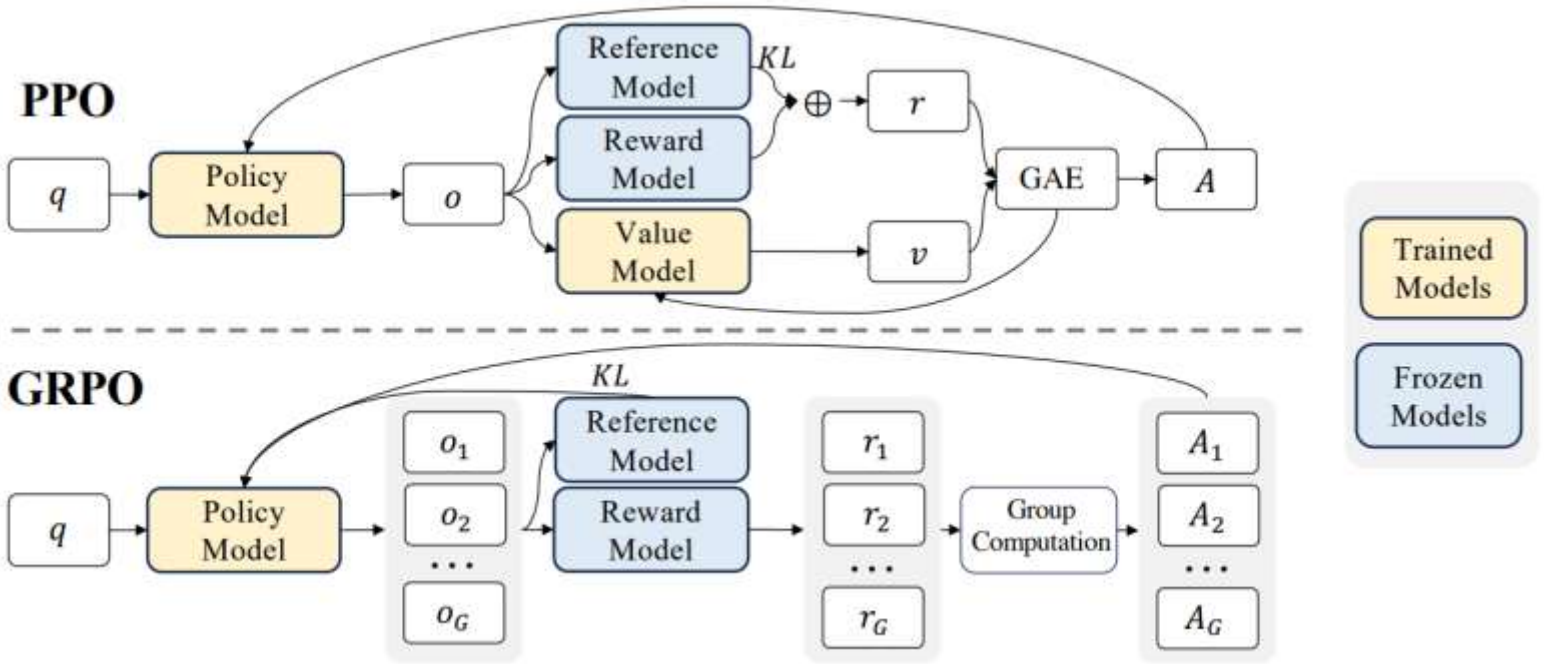
Given a rule or a gold answer,

$$r = R(s, a)$$
$$= \begin{cases} 0, & if\ a\ is\ wrong \\ 1, & if\ a\ is\ right \end{cases}$$

Format rewards:

enforces the model to put its thinking process between ‘<think>’ and ‘</think>’ tags.

Algorithm - GRPO



$$\mathcal{J}_{PPO}(\theta) = \mathbb{E}[q \sim P(Q), o \sim \pi_{\theta_{old}}(O|q)] \frac{1}{|o|} \sum_{t=1}^{|o|} \min\left[\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})} A_t, \text{clip}\left(\frac{\pi_\theta(o_t|q,o_{<t})}{\pi_{\theta_{old}}(o_t|q,o_{<t})}, 1-\varepsilon, 1+\varepsilon\right) A_t\right]$$

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}[q \sim P(Q), \{o_i\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)]$$

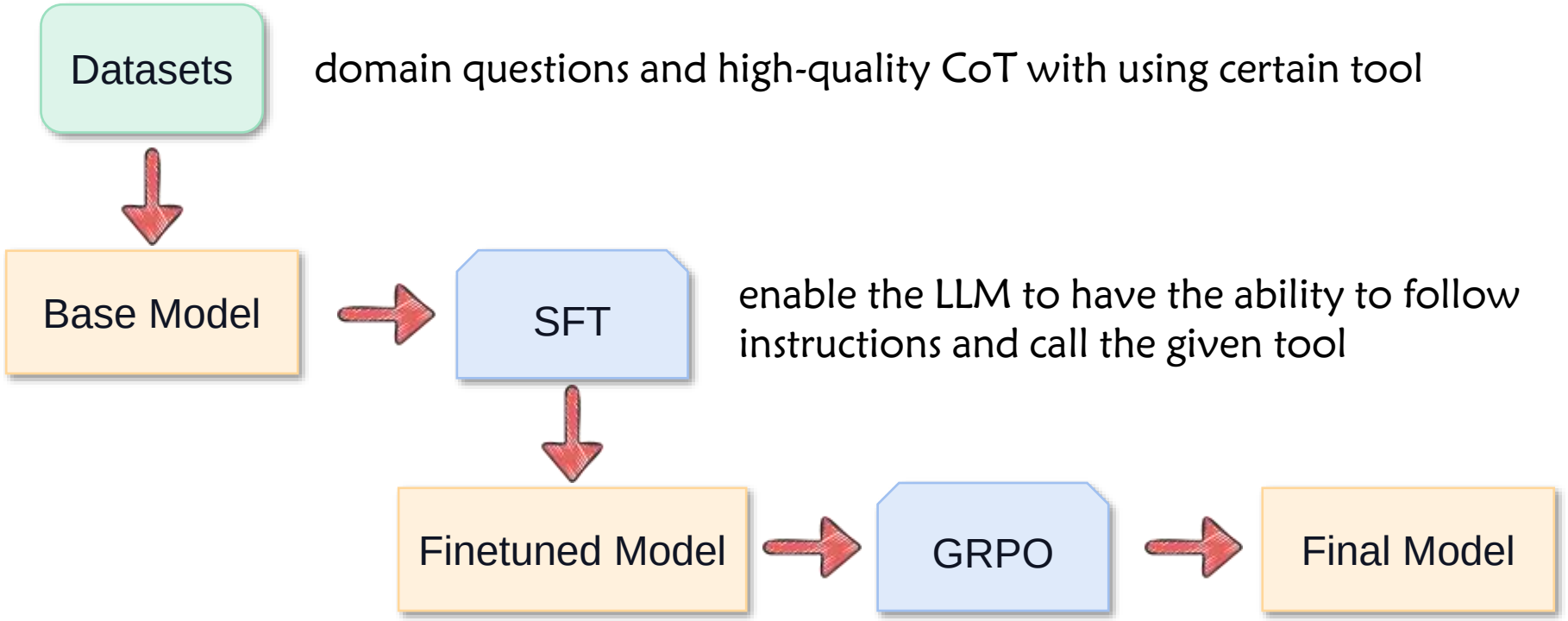
$$\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}\hat{A}_{i,t}, \text{clip}\left(\frac{\pi_\theta(o_{i,t}|q,o_{i,<t})}{\pi_{\theta_{old}}(o_{i,t}|q,o_{i,<t})}, 1-\varepsilon, 1+\varepsilon\right)\hat{A}_{i,t}\right] - \beta\mathbb{D}_{KL}\left[\pi_\theta||\pi_{ref}\right]\right\}$$

$$\hat{A}_{i,t} = \tilde{r}_i = \frac{r_i - \text{mean}(\mathbf{r})}{\text{std}(\mathbf{r})}$$

✓ **lower memory requirements and higher computational efficiency**

Broader Applications

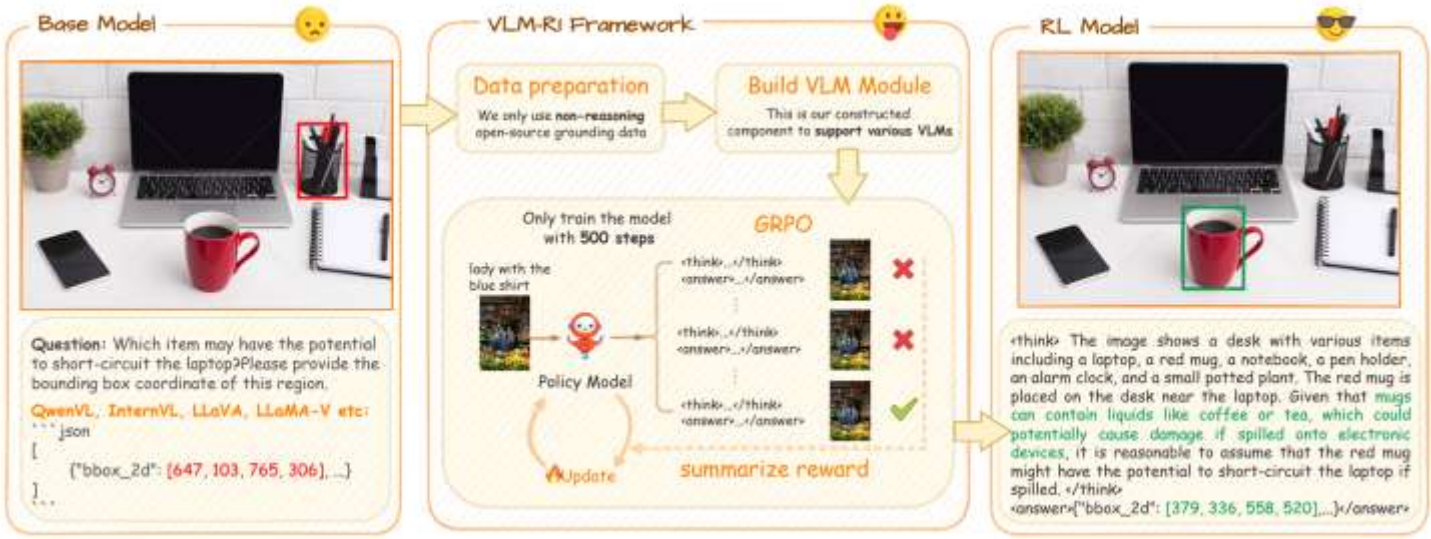
Tool-integrated reasoning



Integrate **external tools** (such as code interpreters and search engines) to enhance the reasoning ability of LLMs

Broader Applications

VLM



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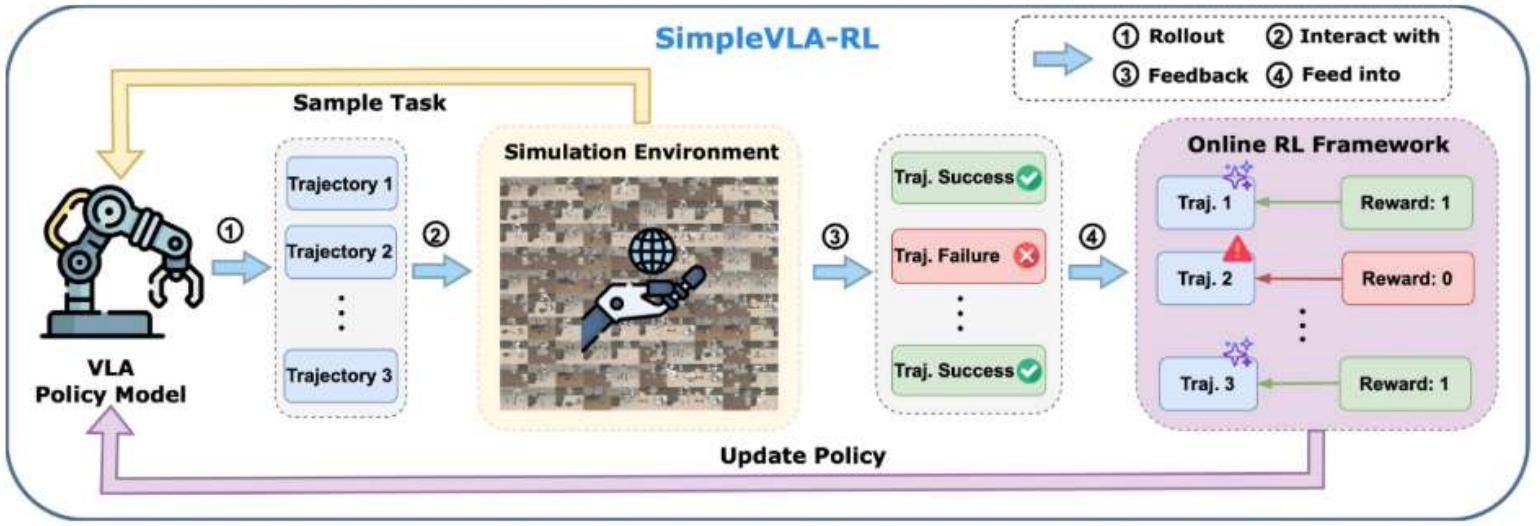
VLA



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The Third Stage - ?

A new generation of agents will acquire superhuman capabilities by learning predominantly from experience.

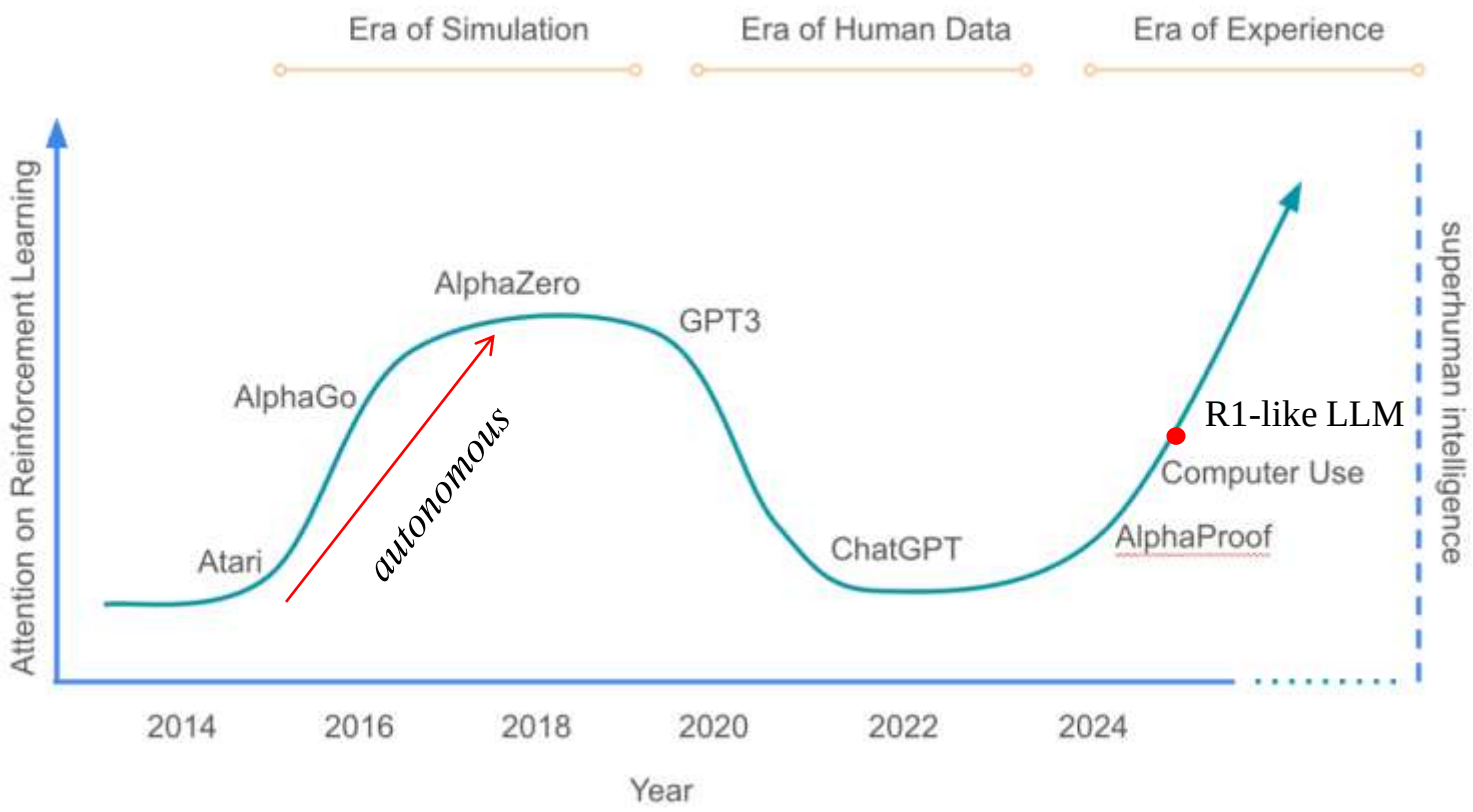
—— David Silver, Richard S. Sutton. *Welcome to the Era of Experience*

- **State Stream**: Shift from "immediate response" to "lifelong learning"

Multi-Turn interaction

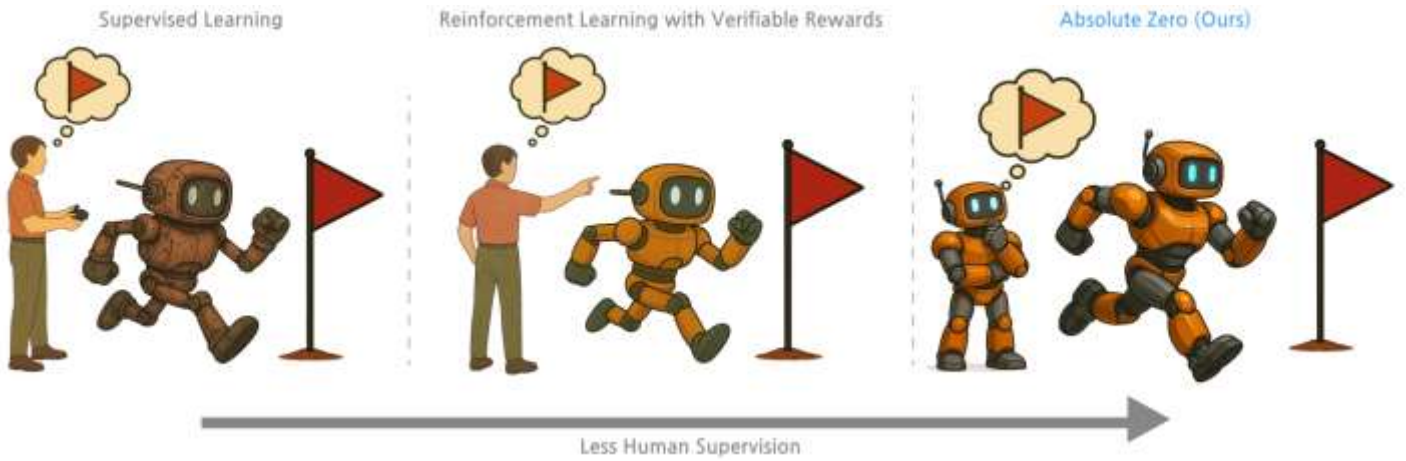
- **Actions and Observations**: Expand the scope of action to achieve autonomous exploration in the real world
- **Rewards**: A hybrid reward mechanism combining grounded signals with dynamic user feedback
- **Planning and Reasoning**: More efficient mechanisms of thought learned from interaction experience

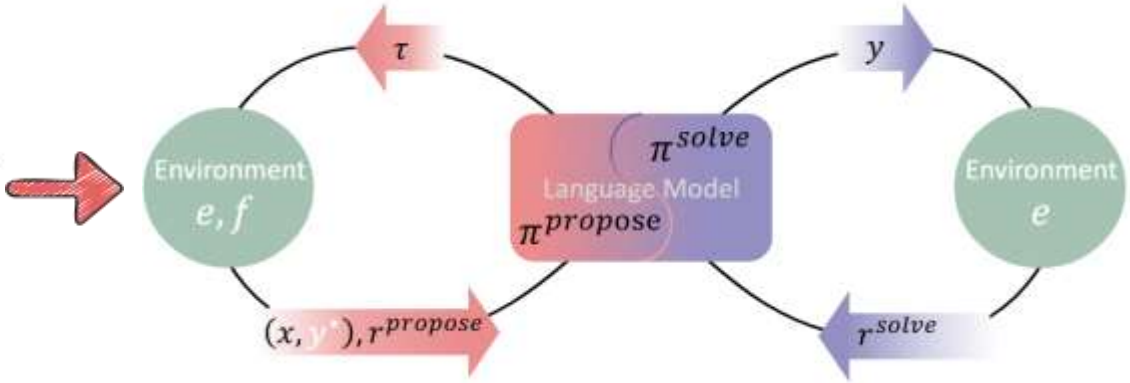
The Third Stage - ?

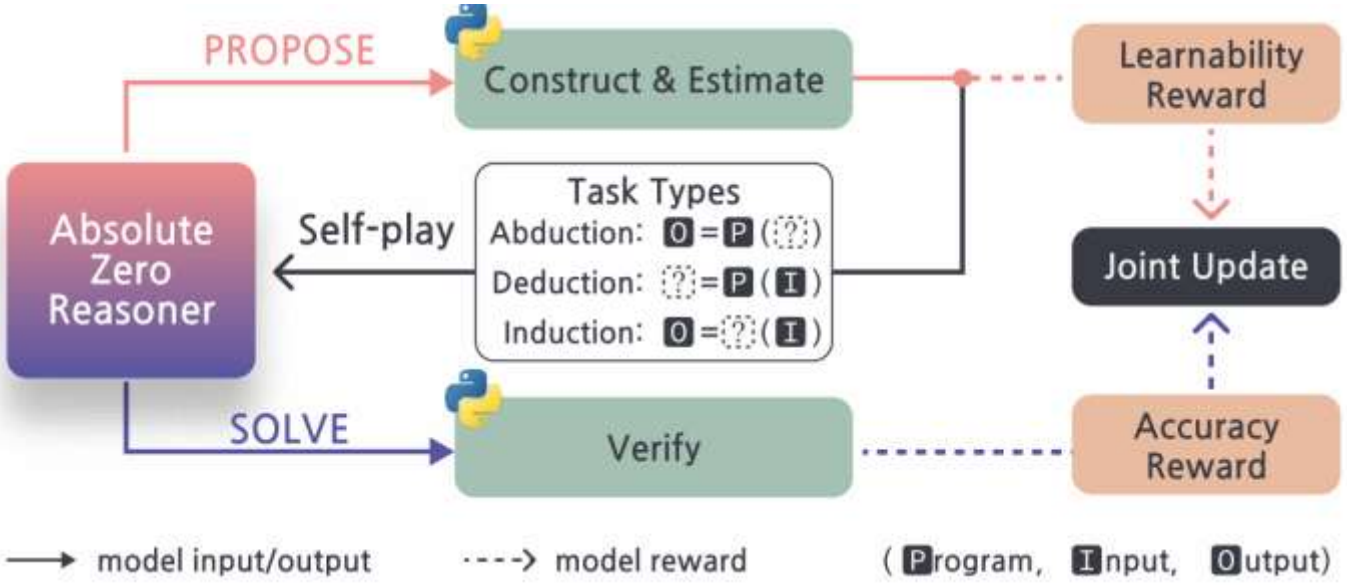


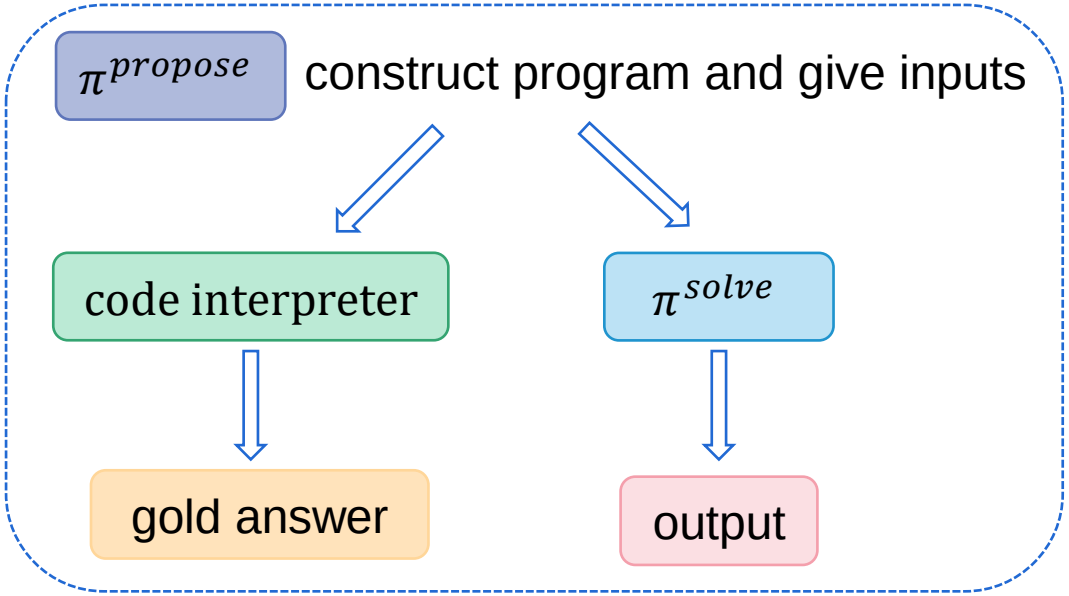
The use of massive amounts of human data as priors has finally enabled RL to generalize

Absolute Zero









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Thanks